Name: Andy Date: 18/11/20

Math 12 Honours Section 1.1 Graphing and Solving Radical Functions

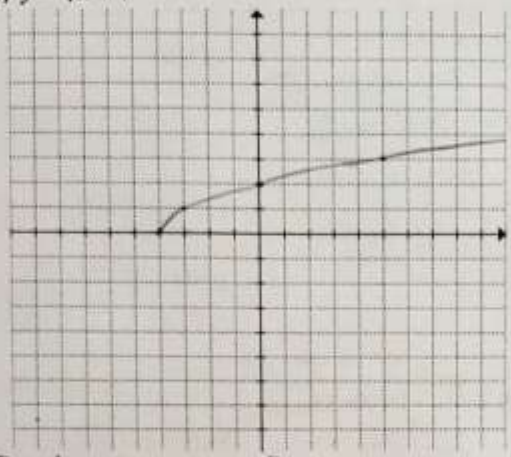
1. What is the domain and Range of the following function? $y = 2\sqrt{3x-4}+2$

   Domain: $x \geq 4/3$

   Range: $y \geq 2$

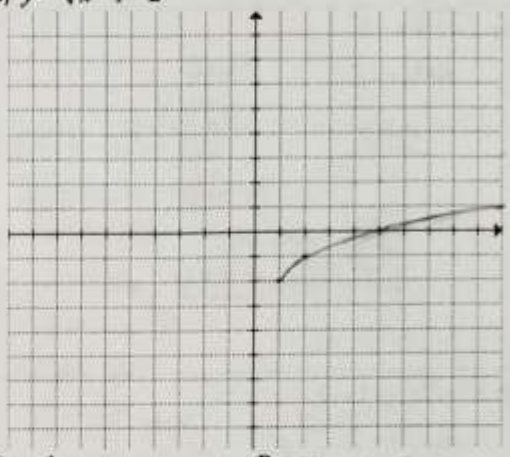
2. Graph each of the following functions below. State the Domain and Range

a) $y = \sqrt{x+4}$



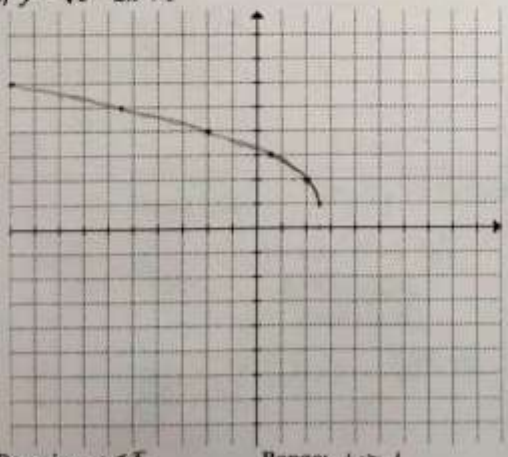
Domain: $x \geq -4$ Range: $y \geq 0$

b) $y = \sqrt{x-1}-2$



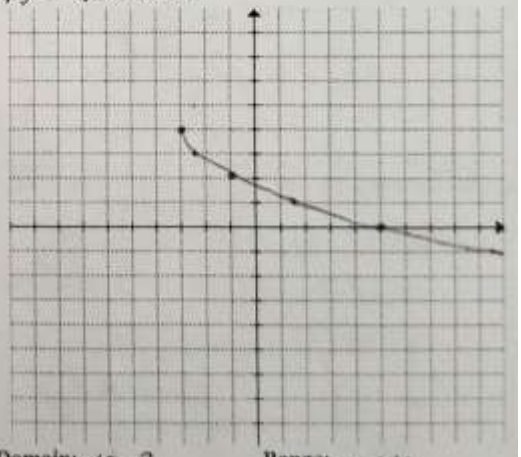
Domain: $x \geq 1$ Range: $y \geq -2$

c) $y = \sqrt{5-2x}+1$



Domain: $x \leq 5/2$ Range: $y \geq 1$

d) $y = -\sqrt{2x+6}+4$

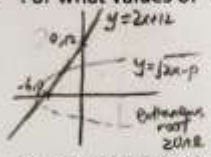


Domain: $x \geq -3$ Range: $y \leq 4$

3. For what values of "k" will the equation have an extraneous root? $\sqrt{3x+5}=k$

$k<0$

4. For what values of "p: will the equation have an extraneous root? $\sqrt{2x-p}=2x+12$



Start of root function: $2x-p\geq 0$, $x\geq \frac{p}{2}$

To have extraneous root, you need the flipped half on side to touch the $2x+12$ line!

so if $\frac{p}{2}<6$ then yes! $p<-12$

5. Solve each of the following equations algebraically. Indicate all extranneous roots:

a) $\sqrt{x+4}=-2$

$x+4=4$

$x=0$

Check: $x=0$ ✗

No actual sols
Extraneous: $x=0$

b) $\sqrt{x+5}=-x-5$

$x+5=x^2+10x+25$

$0=x^2+9x+20$

$0=(x+4)(x+5)$

$x=-4/-5$

Check:
$1=-1$ ✗
$0=0$ ✓

$x=-5$
extraneous: $x=-4$

c) $5-\sqrt{2x-11}=3$

$\sqrt{2x-11}=2$

$2x-11=4$

$x=\frac{15}{2}$

Check: $5-2=3$ ✓

$x=\frac{15}{2}$

d) $2\sqrt{4x-1}+8=16$

$\sqrt{4x-1}=4$

$4x-1=16$

$x=\frac{17}{4}$

Check: $2(4)+8=16$ ✓

$x=\frac{17}{4}$

e) $\sqrt{1+9x}+6=2x$

$1+9x=4x^2-24x+36$

$0=4x^2-33x+35$

$x=\frac{33\pm 23}{8}$

$x=7/\frac{5}{4}$

Check:
$14=14$ ✓
$\frac{19}{2}=\frac{5}{2}$ ✗

$x=7$
Extraneous: $x=\frac{5}{4}$

f) $\sqrt{7x^2-1}+1=3x$

$7x^2-1=9x^2-6x+1$

$0=2x^2-6x+2$

$0=x^2-3x+1$

$x=\frac{3\pm\sqrt{5}}{2}$

Check:
$2.618=2.618$ ✓
$0.381=0.381$ ✓

$x=\frac{3\pm\sqrt{5}}{2}$

g) $\sqrt{3x+10}-\sqrt{5x}=0$

$\sqrt{3x+10}=\sqrt{5x}$

$3x+10=5x$

$10=2x$

$x=5$

Check:

$5-5=0$ ✓

$\boxed{x=5}$

h) $\sqrt{5x^2-2x}=4$

$5x^2-2x=16$

$5x^2-2x-16=0$

$\frac{1}{5}$ ~~$-\frac{2}{8}$~~

$(x-2)(5x+8)=0$

$x=2/-\frac{8}{5}$

Check:

$\sqrt{20-4}=4$ ✓

no

$\boxed{x=2}$

extraneous: $x=-\frac{8}{5}$

e) $\sqrt{x-2}=\sqrt{3x+4}$

$x-2=3x+4$

$2=2x$

$x=1$

Check:

$\sqrt{-1}=\sqrt{7}$ ✗

No real solutions

Extraneous: $x=1$

f) $\sqrt{3x+8}=3\sqrt{x}-2\sqrt{2}$

$3x+8=9x-12\sqrt{2x}+8$

$-6x=-12\sqrt{2x}$

$\frac{1}{2}x=\sqrt{2x}$

$\frac{1}{4}x^2-2x=0$

$x(\frac{1}{4}x-2)=0$

$x=0/8$

Check

$\sqrt{8}=-2\sqrt{2}$ ✗

$\sqrt{32}=3\sqrt{8}-2\sqrt{2}$ ✓

$\boxed{x=8}$

Extraneous: $x=0$

g) $2+\sqrt{x-5}=\sqrt{2x-3}$

$4+4\sqrt{x-5}+x-5=2x-3$

$4\sqrt{x-5}=x-2$

$16x-80=x^2-4x+4$

$0=x^2-20x+84$

-6 -14

$x=6/14$

Check

$2+1=3$ ✓

$2+3=5$ ✓

$\boxed{x=6/14}$

h) $\sqrt{4-x}+\sqrt{x-9}=\sqrt{x-14}$

$4-x+2\sqrt{(4-x)(x-9)}+x-9=x-14$

$2\sqrt{4x-36-x^2+9x}=x-9$

$52x-144-4x^2=x^2-18x+81$

$0=5x^2-70x+225$

$x=\frac{70\pm20}{10}$

$x=9/5$

Check:

No?

No?

No real solutions

Extraneous: $x=9/5$

i) $\frac{2}{\sqrt{x+1}} = \sqrt{x} + \sqrt{x+1}$

$2 = \sqrt{x^2+x} + x + 1$

Check: $\sqrt{3} = \sqrt{1}$ ✓

$\sqrt{x^2+x} = 1 - x$

$x^2 + x = 1 - 2x + x^2$

$x = 1/3$

$\boxed{x = 1/3}$

ii) $\frac{3}{\sqrt{x}} - 5 = \frac{1-2\sqrt{x}}{\sqrt{x}}$

$3 - 5\sqrt{x} = 1 - 2\sqrt{x}$

$2 = 3\sqrt{x}$

$x = \frac{4}{9}$

Check: $\frac{3}{2} - 5 = -\frac{1}{2}$ ✓

$\boxed{x = 4/9}$

6. Solve for "x" $\sqrt{x+\sqrt{x+11}} + \sqrt{x-\sqrt{x+11}} = 4$

$\sqrt{x+A} + \sqrt{x-A} = 4$

$\sqrt{x+A} = 4 - \sqrt{x-A}$

$x + A = 16 - 8\sqrt{x-A} + x - A$

$-\frac{2A+16}{8} = \sqrt{x-A}$

$-\frac{1}{4}A + 2 = \sqrt{x-A}$

$\frac{1}{16}A^2 - A + 4 = x - A$

$\frac{1}{16}(x+11) + 4 = x$

$\frac{75}{16} = \frac{15}{16}x$

$x = 5$

7. Determine the sum of the solution for "x": $\sqrt{2x-7} = 2 + \sqrt{x-7}$

$2x - 7 = 4 + 4\sqrt{x-7} + x - 7$

$x - 4 = 4\sqrt{x-7}$

$x^2 - 8x + 16 = 16x - 112$

$x^2 - 24x + 128 = 0$

$(x-16)(x-8) = 0$

$x = 16, 8$

sum = 24

1. If $\sqrt{4+x} + \sqrt{10-x} = 6$, what is the value of $\sqrt{(4+x)(10-x)}$?

Square it!

$4 + x + 10 - x + 2\sqrt{(4+x)(10-x)} = 36$

$\sqrt{(4+x)(10-x)} = 11$

2. If both "a" and "b" are positive integers, the equation $\sqrt{10} = \sqrt{a} + \sqrt{b}$ has no solutions. For how many positive integral values of $x \le 1000$ does the equation have atleast one solution in positive integers? $\sqrt{x} = \sqrt{a} + \sqrt{b}$

$x = a + b + 2\sqrt{ab}$

So if $\sqrt{ab}$ is a square, then yes!

see page 136

#2

① Square both ends

After squaring both sides, we get $x = a + 2\sqrt{ab} + b$. For x to be an integer, then $\sqrt{ab}$ must be an integer too, or ab is a perfect square.

② GtC $\sqrt{ab}$

$\sqrt{ab} = 1 \rightarrow ab = 1$

a	b	x
1	1	4

$x = 4$

$\sqrt{ab} = 2 \rightarrow ab = 4$

a	b	x
1	4	9
2	2	8
4	1	9

$x = 9/8$

$\sqrt{ab} = 3 \rightarrow ab = 9$

a	b	x
1	9	16
3	3	12
9	1	16

$x = 16/12$

$\sqrt{ab} = 4 \rightarrow ab = 16$

a	b	x
1	16	25
2	8	18
4	4	20
8	2	18
16	1	25

$x = 18/20/25$

Notice how

- There doesn't seem to be overlapping x values. I shall assume there are none.
- The max value of x for each $\sqrt{ab}$ is the next square, if $\sqrt{ab} = 5$, max $x = 36$ for example
  - This means that we only check up to $\sqrt{ab} = 30 \rightarrow x = 31^2 = 961$. Anymore and $x > 1000$ (only check 31 partially)
- The number of values of x per $\sqrt{ab}$ seem to be related to the number of factors of ab (of each square). Looks like for each thing, # of x values $= \frac{\text{factors of } n^2 - 1}{2} + 1$. ($-1 + 1$ comes from the non-factor pair not having a duplicate) This becomes $\frac{\#F + 1}{2}$.
  - E.g. for $\sqrt{ab} = 4$, # of $x = \frac{5+1}{2} = 3$

③ We got a formula!

$$\#\text{ of } x = \sum_{k=1}^{30} \frac{\text{Factors of } k^2 + 1}{2}$$

n^2	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25
#	1	2	2	3	2	5	2	4	3	5	2	8	2	5	5	5	2	8	2	8	5	5	2	11	3

26	27	28	29	30	31
5	4	8	2	14	1

1 961

31 21

$T + 1 = 136$

Name: Andy Date: 19/11/20

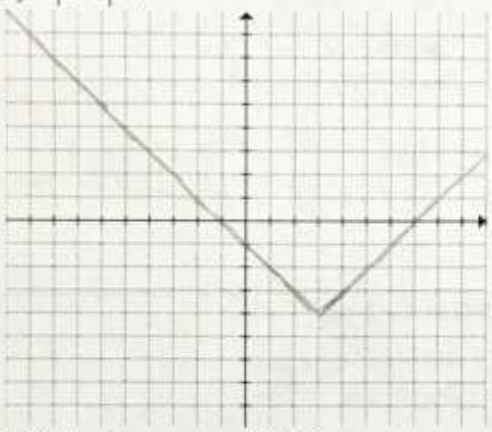
HW Section 1.2 Graphing and Solving Absolute Value Functions:

1. Given the equation, for what values of "k" will it have an extraneous root? $|3x+11|-11=k$

$|3x+11|=k+11$ $k<-11$ (this is not because negative!)

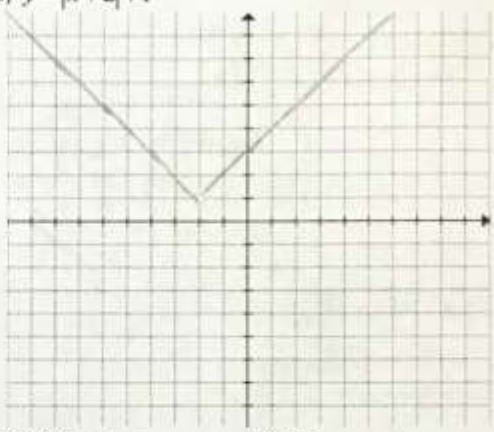
2. Graph each of the following functions with the grid provided. State the Domain and Range:

a) $y=|x-3|-4$



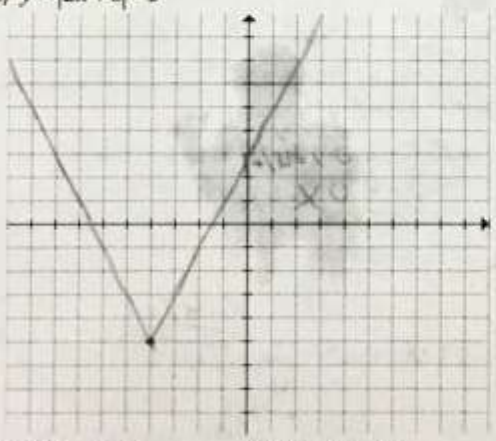
Domain: $x \in R$ Range: $y \geq -4$

b) $y=|x+2|+1$



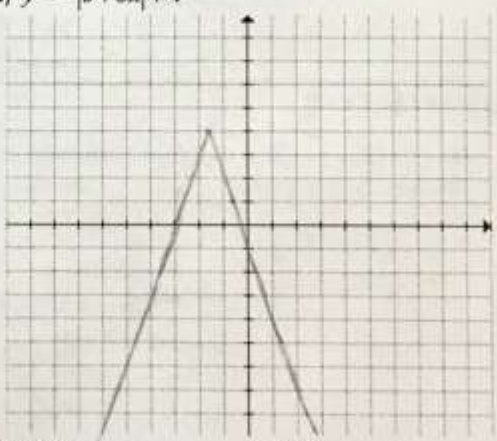
Domain: $x \in R$ Range: $y \geq 1$

c) $y=|2x+8|-5$



Domain: $x \in R$ Range: $y \geq -5$

d) $y=-|5+3x|+4$



Domain: $x \in R$ Range: $y \leq 4$

3. Solve the following equations and indicate any extranneous roots:

a) $|-2x| = 20$

$-2x = 20 \quad x = -10$ ✓

$2x = 20 \quad x = 10$ ✓

$x = -10/10$

b) $|8x - 15| = 7$

$8x - 15 = 7 \quad x = \frac{11}{4}$ ✓

$8x - 15 = -7 \quad x = 1$ ✓

$x = 1/\frac{11}{4}$

c) $|-9x - 20| + 23 = 0$

$|-9x - 20| = -23$

$-9x - 20 = -23 \quad x = \frac{1}{3}$ X

$-9x - 20 = 23 \quad x = -\frac{43}{9}$ X

No real sols

Extraneous: $\frac{1}{3}, -\frac{43}{9}$

d) $\sqrt{(2x-13)^2} = x + 4$

$|2x - 13| = x + 4$

$2x - 13 = x + 4 \quad x = 17$ ✓

$2x - 13 = -x - 4 \quad x = 3$ ✓

$x = 3/17$

e) $|3x + 14| = x + 7$

$3x + 14 = x + 7 \quad x = -\frac{7}{2}$ ✓

$3x + 14 = -x - 7 \quad x = -\frac{21}{4}$ ✓

$x = -\frac{7}{2}/-\frac{21}{4}$

f) $|2x + 1| = 3 - x$

$2x + 1 = 3 - x \quad x = \frac{2}{3}$ ✓

$2x + 1 = -3 + x \quad x = -4$ ✓

$x = \frac{2}{3}/-4$

g) $|4x + 11| = 2x - 3$

$4x + 11 = 2x - 3 \quad x = -7$ X

$4x + 11 = -2x + 3 \quad x = -\frac{4}{3}$ X

No real sols

Extraneous: $-7, -\frac{4}{3}$

h) $|x^2 - 3x| = 4$

$x^2 - 3x - 4 = 0 \quad x = 4/-1$

$x^2 - 3x + 4 = 0$ X

$x = 4/-1$

i) $|13x - x^2| = 30$

$13x - x^2 = 30$ $x^2 - 13x + 30 = 0$ $(x-10)(x-3) = 0$

$13x - x^2 = -30$ $x^2 - 13x - 30 = 0$ $(x-15)(x+2) = 0$

$x = 10/3/15/-2$

✓ ✓ ✓ ✓

$x = 10/3/15/-2$

j) $|2x^2 - x - 6| = 2x + 1$

$2x^2 - x - 6 = 2x + 1$ $2x^2 - 3x - 7 = 0$

$2x^2 - x - 6 = -2x - 1$ $2x^2 + x - 5 = 0$

$x = \frac{3 \pm \sqrt{65}}{4} / \frac{-1 \pm \sqrt{41}}{4}$

✓ ✗ ✓ ✗

$x = \frac{3+\sqrt{65}}{4} / \frac{-1+\sqrt{41}}{4}$

extraneous: $\frac{3-\sqrt{65}}{4} / \frac{-1-\sqrt{41}}{4}$

k) $|4x + 10| = 2x^2 + 1$

$4x + 10 = 2x^2 + 1$ $2x^2 - 4x - 9 = 0$ $x = \frac{2 \pm \sqrt{22}}{2}$ ✓✓

$-4x - 10 = 2x^2 + 1$ $2x^2 + 4x + 11 = 0$ $x = -4 \pm \sqrt{\ }$ ✗

$x = \frac{2 \pm \sqrt{22}}{2}$

l) $|x^2 - 4| = 2x^2 - 3$

$x^2 - 4 = 2x^2 - 3$ $x^2 = -1$ ✗

$-x^2 + 4 = 2x^2 - 3$ $x = \pm\sqrt{\frac{7}{3}} = \pm\frac{\sqrt{21}}{3}$ ✓✓

$x = \pm\frac{\sqrt{21}}{3}$

4. Solve for "x": $|x+4| = |-12|$

$|x+4| = 12$

$x + 4 = 12$ $x = 8$ ✓

$x + 4 = -12$ $x = -16$ ✓

$x = 8/-16$

5. For what values of "k" will the solution have 3 or more solutions? $|10x + x^2| = k$

$10x + x^2 - k = 0$

$10x + x^2 + k = 0$

Discriminant of both

$100 + 40k \quad >0 \quad =0 \quad <0$

$100 - 40k \quad >0 \quad =0 \quad <0$

$2 \quad 1 \quad 0$

>0 $=0$ $k = \frac{5}{2}$

$=0$ >0 $k = -\frac{5}{2}$

$100 > -40k \quad -\frac{5}{2} < k$

$100 > 40k \quad \frac{5}{2} > k$

$-\frac{5}{2} \le k \le \frac{5}{2}$

6. Solve for all real numbers "x" such that $|(5 - |x|)| \le 14$

++ $5 - x \le 14$ $x \ge -9$

+- $5 + x \le 14$ $x \le 9$

-+ $5 - x \le -14$ $x \ge 19$

-- $5 + x \le -14$ $x \le -19$

$-19 \le x \le 19$

7. How many solutions does the following have? Find all the solutions: $|x^2-12|=0.5x+4$

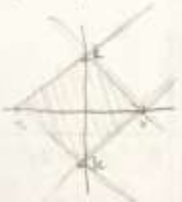
$x^2-12=\frac{1}{2}x+4 \quad x^2-\frac{1}{2}x-16=0 \quad 2x^2-x-32=0 \quad x=\frac{1\pm\sqrt{257}}{4}$ ✓✓

$x^2-12=-\frac{1}{2}x-4 \quad x^2+\frac{1}{2}x-8=0 \quad 2x^2+x-16=0 \quad x=\frac{-1\pm\sqrt{129}}{4}$ ✓✓

3 sols – $\frac{1\pm\sqrt{257}}{4}, \frac{-1\pm\sqrt{129}}{4}$

8. The graphs of $|x+y|=k$ and $|x-y|=k$ creates a square. What is the value of "k" is the area of this square is 20?

$x+y=k \rightarrow y=-x+k$
$-x-y=k \rightarrow y=-x-k$
$x-y=k \rightarrow y=x-k$
$-x+y=k \rightarrow y=x+k$



$4\times\frac{1}{2}\times k^2=20$

$k=\pm\sqrt{10}$

9. How many integers satisfy the following: $|x|+1\geq 3$ *and* $|x-1|<3$?

$|x|+1\geq 3$
$x+1=3 \quad x=2$
$-x+1=3 \quad x=-2$
$x\leq -2 \quad x\geq 2$

$|x-1|<3$
$x-1=3 \quad x=4$
$x-1=-3 \quad x=-2$
$-2<x<4$

$x=2,3$

so 2

10. What is the domain and range of the following graph: $|x^2-y^2|=3$ and $|x^2+y^2|=3$

$|x^2-y^2|=3$
① $x^2-y^2=3$
② $x^2-y^2=-3$

① $y=\pm\sqrt{x^2-3}$ D: $x\in R$ R: $y\in R$
② $y=\pm\sqrt{x^2+3}$

$|x^2+y^2|=3$
$x^2+y^2=3$

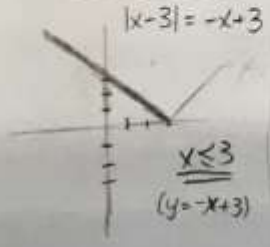
(Thank you classmates)
D: $-\sqrt{3}\leq x\leq\sqrt{3}$
R: $-\sqrt{3}\leq y\leq\sqrt{3}$

10 Challenge: The graph of the three equations are given in the coordinate plane. How mnay ordered pairs of integers (x,y) satisfy all three equations? List all the coordinate pairs: ....

$y-|y|=0 \quad ; \quad x-3+|x-3|=0$ and $y-x+|y-x|=0$

$y-|y|=0$
$y=|y|$
$y\geq 0$

$x-3+|x-3|=0$
$|x-3|=-x+3$



$x\leq 3$
$(y=-x+3)$

$y-x+|y-x|=0$
$y-x+|y-x|=0 \rightarrow y=x$
$y-x-y+x=0 \rightarrow ?$

Doing a plot gives
$y\leq x$

So $y\geq 0, x\leq 3, y\leq x$

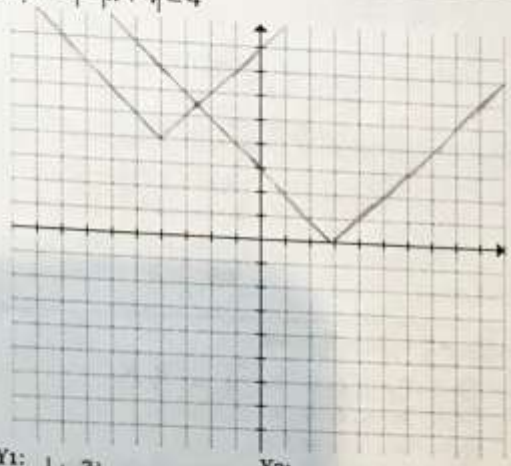
(0,0)
(1,0)(1,1)
(2,0)(2,1)(2,2)
(3,0)(3,1)(3,2)(3,3)

Name: Andy Date: 20/11/20

HW Section 1.3 Solving Systems with Two or More Absolute Value Functions:

1. Given each equation, indicate how you will separate "y1" and "y2". Then graph each function separately to find the intersection point:

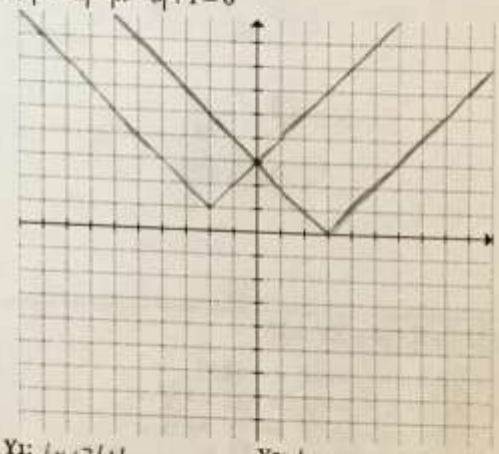
a) $|x-3|-|x+4|=4$



Y1: $|x-3|$ Y2: $4+|x+4|$

Intersection Points: $\left(-\frac{5}{2}, \frac{11}{2}\right)$

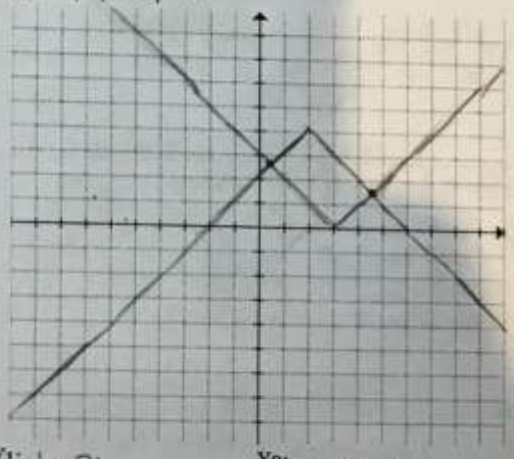
b) $|x+2|-|x-3|+1=0$



Y1: $|x+2|+1$ Y2: $|x-3|$

Intersection Points: $(0,3)$

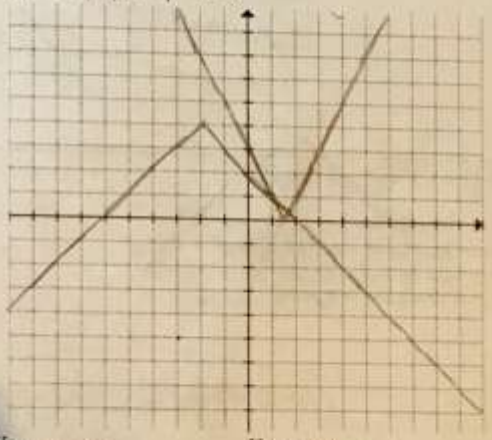
c) $|x-3|+|x-2|=4$



Y1: $|x-3|$ Y2: $4-|x-2|$

Intersection Points: $\left(\frac{1}{2}, \frac{5}{2}\right)$ $\left(\frac{9}{2}, \frac{3}{2}\right)$

d) $|2x-3|+|x+2|-4=0$



Y1: $|2x-3|$ Y2: $4-|x+2|$

Intersection Points: $(1,1)$ $\left(\frac{5}{3}, \frac{1}{3}\right)$

2. Solve the following equations and indicate any extranneous roots:

e) $|x+3|+|x-6|=16$

++ $x+3+x-6=16$ $2x=19$ $x=\frac{19}{2}$ ✓
+− $x+3-x+6=16$ NO
−+ $-x-3+x-6=16$ NO
−− $-x-3-x+6=16$ $2x=-13$ $x=-\frac{13}{2}$ ✓

$x=\frac{19}{2}/-\frac{13}{2}$

f) $|x+3|+|5-x|=16$

++ $x+3+5-x=16$ NO
+− $x+3-5+x=16$ $x=9$ ✓
−+ $-x-3+5-x=16$ $x=-7$ ✓
−− $-x-3-5+x=16$ NO

$x=9/-7$

g) $|2x+1|+|4-3x|=18$

++ $2x+1+4-3x=18$ $x=-13$ ✗
+− $2x+1-4+3x=18$ $5x=21$ $x=\frac{21}{5}$ ✓
−+ $-2x-1+4-3x=18$ $-5x=15$ $x=-3$ ✓
−− $-2x-1-4+3x=18$ $x=22$ ✗

$x=-3/2\frac{1}{5}$
Extraneous: $x=-13/22$

h) $|3x-1|+|3x-5|=-4$

++ $3x-1+3x-5=-4$ $6x=2$ $x=\frac{1}{3}$ ✗
+− $3x-1-3x+5=-4$ NO
−+ $-3x+1+3x-5=-4$ NO
−− $-3x+1-3x+5=-4$ $-6x=-10$ $x=\frac{5}{3}$ ✗

No real sols
Extraneous: $x=\frac{1}{3}/\frac{5}{3}$

i) $|2x+3|+|4-3x|=15$

++ $2x+3+4-3x=15$ $x=-8$ ✗
+− $2x+3-4+3x=15$ $5x=16$ $x=\frac{16}{5}$ ✓
−+ $-2x-3+4-3x=15$ $-5x=14$ $x=-\frac{14}{5}$ ✓
−− $-2x-3-4+3x=15$ $x=23$ ✗

$x=\frac{-14}{5}/\frac{16}{5}$
Extraneous: $x=-8/23$

j) $|2x+3|+|3x-8|=15$

++ $2x+3+3x-8=15$ $5x=20$ $x=4$ ✓
+− $2x+3-3x+8=15$ $x=-4$ ✗
−+ $-2x-3+3x-8=15$ $x=26$ ✗
−− $-2x-3-3x+8=15$ $-5x=10$ $x=-2$ ✓

$x=-2/4$
Extraneous: $x=-4/26$

3. Given the following equation, is the solution a set of points or a range of values? Explain:

$|2x-1|+|2x-5|-4=0$

Since the 0=0 exists, we have a range somewhere.

$2x-1+2x-5-4=0$ $4x=10$ $x=5/2$ ✓

$2x+1-2x+5-4=0$ $0=0$!

$-2x+1+2x-5-4=0$ $-8=0$ ✗

$-2x+1-2x+5-4=0$ $-4x=-2$ $x=1/2$ ✓

Graph:

$\frac{1}{2} \le x \le \frac{5}{2}$

4. Find all the value(s) of "x" for which the equation is true: $|x|=|x+1|$

++ $x=x+1$ NO

+− $x=-x-1$ $x=-\frac{1}{2}$ ✓

−+ $-x=x+1$ $x=-\frac{1}{2}$ ✓

−− $-x=-x-1$ NO

$x=-\frac{1}{2}$

5. Find the two value(s) that will satisfy the equation: $|x-1|+|x|+|x+1|=\frac{5}{2}$

+++ $x-1+x+x+1=\frac{5}{2}$

++− $x-1+x-x-1=\frac{5}{2}$

+−+ $x-1-x+x+1=\frac{5}{2}$

+−− $x-1-x-x-1=\frac{5}{2}$

$x=5/6/\frac{9}{4}/\frac{5}{2}/-\frac{9}{2}$

× × × ×

−++ $-x+1+x+x+1=\frac{5}{2}$

−+− $-x+1+x-x-1=\frac{5}{2}$

−−+ $-x+1-x+x+1=\frac{5}{2}$

−−− $-x+1-x-x-1=\frac{5}{2}$

$x=\frac{1}{2}/-\frac{5}{2}/-\frac{1}{2}/-\frac{5}{6}$

✓ × ✓ ×

$x=\frac{1}{2}/-\frac{1}{2}$

6. Solve for "x" $|x^2-9x+20|=|16-x^2|$

++ $x^2-9x+20=16-x^2$ $2x^2-9x+4=0$

+− $x^2-9x+20=-16+x^2$

−+ $-x^2+9x-20=16-x^2$

−− $-x^2+9x-20=-16+x^2$ $2x^2-9x+4=0$

$(x-4)(2x-1)=0$

$x=4/\frac{1}{2}$

✓ ✓

$x=\frac{1}{2}/4$

7. How many ordered pairs of integers (a,b) satisfy this equation? $|a-2|\times|b-3|=2$

Integers → divisors!

1 2 → $a=3/1$ $b=5/1$

2 1 → $a=1/0$ $b=4/2$

−1 −2 → NO. ($ab \neq 0$)

−2 −1 → NO ($ab \neq 0$)

(3,5)(1,1)
(1,4)(0,2)

4

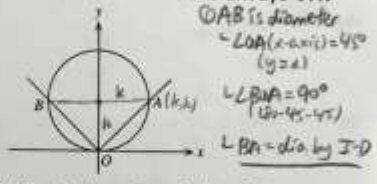
8. What is the smallest value of "x" such that $|5x-1|=|3x+2|$? Express your answer as a common fraction

$5x-1=3x+2 \quad 2x=3 \quad x=3/2$ ✓
$5x-1=-3x-2 \quad 8x=-1 \quad x=-1/8$ ✓
$-5x+1=3x+2 \quad 8x=-1 \quad x=-1/8$
$-5x+1=-3x-2 \quad -2x=-3 \quad x=3/2$

$x=-1/8, 3/2$

so smallest is $x=-1/8$

9. A circle with its center on the y-axis intersects the graph of $y=|x|$ at the origin and exactly two other points "A" and "B" as shown in the diagram below. Prove that the ratio of the area of triangle ABO to the area of the circle is always $1:\pi$

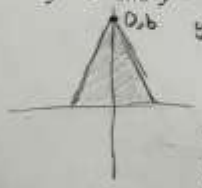


① AB is diameter
∠OA(x-axis) = 45° (y=x)
∠BOA = 90° (180-45-45)
∠BA = dia by I-D

② Let A = (k,k)
△BAO: $\frac{1}{2} \times k \times 2k = k^2$
○: $k^2\pi$

③ △:○ = $1:\pi$ QED

10. What is the value of $b>0$ for which the region bounded by both equations has an area of 72?
$y=0$ and $y=-|2x|+b$?



$y=-|2x|+b$ becomes
$y=-2x+b$
$y=2x+b$
Top point = (0,b)
y-int's = $-\frac{b}{2}, \frac{b}{2}$

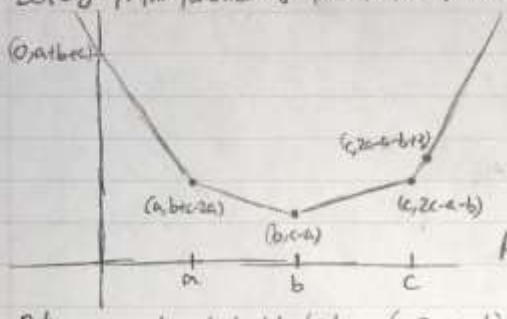
$b \times b \times \frac{1}{2} = 72$
$b^2 = 144$
$b=12$

11. Challenge: Positive integers "a", "b", and "c" are chosen so that $a<b<c$, and the system of equation: $2x+y=2003$ and $y=|x-a|+|x-b|+|x-c|$ has exactly one solution. What is the minimum value of "c"?

1002 see ppr for an answer
I 100% ripped off

#11 (stolen completely off AOPS) (sorry)

① Let's graph this function $y = |x-a| + |x-b| + |x-c|$, using a, b and c as the dividers.



The y-intercept is $(0, a+b+c)$

At $x=a$, $y = |a-a| + |a-b| + |a-c|$

$= b-a+c-a$, so $(a, b+c-2a)$

slope $= \frac{b+c-2a-a-b+c}{a-0} = -3$

At $x=b$, $y = |b-a| + |0| + |b-c|$, so $(b, c-a)$

slope $= \frac{b-c+2a}{c-a-a} = -1$

At $x=c$, $y = |c-a| + |c-b| + |0|$, so $(c, 2c-a-b)$. Slope $= \frac{2c-a-b-c+a}{c-b} = 1$

At $x>c$, let's just use $c+1$. This becomes $(c+1, 2c-a-b+3)$. So, a slope of 3

② So, it forms a $\vee$ thing! All we need now is to find the 1 intersection!

After a bit of thought, the only way for $y = -2x+2003$ to intersect this function at 1 point is at $(a, b+c-2a)$

Any more to the left, and the line might also intercept the $a \le x \le b$ segment

Any more to the right, and intercept the $a \le x \le c$ whatever other point it ends up on

So, the intersection point is $(a, b+c-2a)$

③ Finding c

$-2a + 2003 = b+c-2a$

$b+c = 2003$

Since both are integral, and $c > b$, then the smallest value of c is 1002 ($b = 1001$)

Name: Andy Date: 23/11/20

HW 1.4 Simplifying Rational Expressions and Finding NPV's

1. Given each algebraic expression, indicate which ones are rational expressions. Explain why or why not:

a) $y=\frac{x+2}{x-3}$ Yep	b) $y=\frac{12}{x}$ Yes	c) $y=\frac{\sqrt{x}}{x}$ NO no radicals allowed
d) $y=\frac{2^x+\sqrt{7}}{x^3+10x}$ NO no powers of x	e) $y=x^{-3}+12x^{-2}$ Yeppers $x^{-3}=\frac{1}{x^3}$	f) $y=\frac{\sqrt{8x^3+2x^2+3}}{x^2-3}$ Sure
g) $y=\frac{\sqrt{x^3}+2x^2+3}{x^2-3}$ NO rational x's	h) $y=16x$ Sure	i) $y=12$ Absolutely N-YES

2. Given each rational expression, find the non-permissible values

a) $\frac{2x}{(x-1)(x+4)}$ $x\neq 1,-4$	b) $\frac{x-1}{x(x-1)(2x+5)}$ $x\neq 0,1,-\frac{5}{2}$	c) $\frac{2x}{(x-3)(2x+7)}$ $x\neq 3,-\frac{7}{2}$
d) $\frac{5x}{(2x-3)+(3x-7)}$ $=\frac{5x}{5x-10}$ $x\neq 2$	e) $\frac{2x}{2x(4x+3)-3(4x+3)}$ $=\frac{2x}{(4x+3)(2x-3)}$ $x\neq -\frac{3}{4},\frac{3}{2}$	f) $\frac{6x}{x^2-2x-24}$ $=\frac{6x}{(x-6)(x+4)}$ $x\neq 6,-4$
g) $\frac{5-2x}{3x^2-11x+6}$ $\frac{1}{3}$ $-\frac{2}{3}$ $\frac{5-2x}{(x-3)(3x-2)}$ $x\neq 3,\frac{2}{3}$	h) $\frac{2x^2-3x}{12x^2-53x+56}$ $\frac{3}{4}$ -7 -8 $\frac{2x^2-3x}{(3x-8)(4x-7)}$ $x\neq \frac{8}{3},\frac{7}{4}$	i) $\frac{8-2x}{2x^3+5x^2-12x}$ $\frac{1}{2}$ $-\frac{3}{4}$ $\frac{8-2x}{x(x+4)(2x-3)}$ $x\neq 0,-4,\frac{3}{2}$
j) $\frac{3}{(x^2-16)(x^2+1)}$ $=\frac{3}{(x-4)(x+4)(x^2+1)}$ $x\neq\pm 4$	k) $\frac{5}{x(x^3-8)(x^2+2)}$ $=\frac{5}{x(x-2)(x^2+2x+4)(x^2+2)}$ $x\neq 0,2$	L) $\frac{2}{x^3+3x^2+3x+1}$ $=\frac{2}{(x+1)^3}$ $x\neq -1$
m) $\frac{2x-3}{x+2}\div\frac{3x+4}{x^2-9}$ $x\neq -2,\pm 3,-\frac{4}{3}$ $=\frac{2x-3}{x+2}\div\frac{3x+4}{(x-3)(x+3)}$ $=\frac{2x-3}{x+2}\times\frac{(x-3)(x+3)}{3x+4}$	n) $\frac{x+14}{x^2-16}\div\frac{x^2-5x+14}{x^2-2x+8}$ $x\neq\pm 4,-2,7$ $=\frac{x+14}{(x-4)(x+4)}\div\frac{(x-7)(x+2)}{(x-4)(x+2)}$ $=\frac{x+14}{(x-4)(x+4)}\times\frac{(x-4)(x+2)}{(x-7)(x+2)}$	O) $\frac{\frac{2}{x}-\frac{3}{x+2}}{\frac{1}{x-2}-\frac{x}{x-1}}=\frac{\frac{-x+4}{x(x+2)}}{\frac{-x^2+3x-1}{(x-2)(x-1)}}$ $=\frac{-x+4}{x(x+2)}\times\frac{(x-2)(x-1)}{-x^2+3x-1}$ NPV: $0,\pm 2,1$

NPV: ok	NPV: uh	oopsies

3. Simplify by Multiplying or Dividing. Indicate all NPV's

a) $\frac{(2x)^2}{5y} \times \frac{10x}{8y} \div \frac{15x}{(4y)^2}$

$= \frac{4x^2}{5y} \times \frac{2 \cdot 5x}{2 \cdot 4y} \times \frac{2 \cdot 8y}{15x}$

$= \frac{16x^2}{15}$

$y \neq 0$
$x \neq 0$

b) $\frac{15x}{2x+6} \div \frac{10x}{3x+9}$

$= \frac{3 \cdot 5x}{2(x+3)} \times \frac{3(x+3)}{5x}$

$= \frac{9}{4}$

$x \neq -3, 0$

c) $\frac{(x+1)^2}{x^2-1} \times \frac{x^2-4}{x^2+3x+2}$

$= \frac{(x+1)^2}{(x-1)(x+1)} \times \frac{(x-2)(x+2)}{(x+1)(x+2)}$

$= \frac{x-2}{x+1}$

$x \neq \pm 1, -2$

d) $\frac{x^2+5x+6}{x^2-5x+6} \div \frac{x^2-x-6}{x^2+x-6}$

$= \frac{(x+2)(x+3)}{(x-2)(x-3)} \div \frac{(x-3)(x+2)}{(x+3)(x-2)}$

$= \frac{(x+2)(x+3)}{(x-2)(x-3)} \times \frac{(x+3)(x-2)}{(x-3)(x+2)}$

$= \frac{(x+3)^2}{(x-3)^2}$

$x \neq \pm 2, \pm 3$

e) $\frac{x^2-16y^2}{6x^2y} \div \frac{x^2+xy-20y^2}{4x^3y^2}$

$= \frac{(x-4y)(x+4y)}{6x^2y} \div \frac{(x+5y)(x-4y)}{4x^3y^2}$

$= \frac{(x-4y)(x+4y)}{3 \cdot 2x^2y} \times \frac{2 \cdot 2x^3y^2}{(x+5y)(x-4y)}$

$= \frac{2(x+4y)}{3(x+5y)}$

$x \neq 0, 4y, -5y$
$y \neq 0$

f) $\frac{x^2+4x-5}{3x-6} \times \frac{x-2}{1-x} = \frac{(x-1)(x+5)(x-2)}{3(x-2)(-)(x-1)}$

$= \frac{-x-5}{3}$

$x \neq 2, 1$

g) $\frac{m^2-9mn+14n^2}{m^2+7mn+12n^2} \div \frac{3m^2-21mn}{4m^3+16m^2n}$

$= \frac{(m-2n)(m-7n)}{(m+3n)(m+4n)} \div \frac{3m(m-7n)}{4m^2(m+4n)}$

$= \frac{(m-2n)(m-7n)}{(m+3n)(m+4n)} \times \frac{4m^2(m+4n)}{3m(m-7n)}$

$= \frac{4(m-2n)}{3(m+3n)}$

$m \neq 0, -3n, -4n, 7n$

h) $\frac{3x^2+3x-6}{x^2y-7xy} \times \frac{x^2y-13xy+42y}{6x^2+12x}$

$= \frac{(x-1)(3x+6)}{x(x-7)} \times \frac{y(x-6)(x-7)}{2 \cdot 3x(x+2)}$

$= \frac{(x-1)(x-6)}{2x^2}$

$x \neq 0, 7, -2$
$y \neq 0$

i) $\frac{x+2y}{x-3y}\times\frac{x^2-9y^2}{x^2-4y^2}\div\frac{x+3y}{x-2y}$

$=\frac{x+2y}{x-3y}\times\frac{(x-3y)(x+3y)}{(x-2y)(x+2y)}\times\frac{x-2y}{x+3y}$

$=1$

$x\neq 3y,\pm 2y$

j) $\frac{(3a+7b)^2}{2a-5b}\times\frac{4a^2-25b^2}{9a^2-49b^2}\div\frac{2a+5b}{3a-7b}$

$=\frac{(3a+7b)^2}{2a-5b}\times\frac{(2a-5b)(2a+5b)}{(3a-7b)(3a+7b)}\times\frac{(3a-7b)}{2a+5b}$

$=3a+7b$

$a\neq\pm\frac{5}{2}b,\pm\frac{7}{3}b$

4. Add or Subtract each of the following expressions below. State all the NPV's

a) $\frac{2}{x+5}+\frac{3}{x+2}$ $x\neq -5,-2$

$=\frac{2x+4+3x+15}{(x+5)(x+2)}$

$=\frac{5x+9}{(x+5)(x+2)}$

b) $\frac{x-2}{x+2}+\frac{x+1}{x-4}$ $x\neq -2,4$

$=\frac{(x-2)(x-4)+(x+1)(x+2)}{(x+2)(x-4)}=\frac{2x^2-3x+10}{(x+2)(x-4)}$

$=\frac{x^2-6x+8+x^2+3x+2}{(x+2)(x-4)}$

c) $\frac{3}{x-1}-\frac{2}{2-x}$ $x\neq 1,2$

$=\frac{6-3x-2x+2}{(x-1)(2-x)}$

$=\frac{-5x+8}{(x-1)(2-x)}$

d) $\frac{5x}{3x+9}-\frac{9x}{2x+6}$ $x\neq -3$

$=\frac{5x(2)-9x(3)}{6(x+3)}$

$=\frac{10x-27x}{6(x+3)}$

$=\frac{-17x}{6(x+3)}$

e) $\frac{2}{x-2}+\frac{2x}{2-x}+\frac{x+2}{x-2}$ $x\neq 2$

$=\frac{2}{x-2}+\frac{-2x}{(x-2)}+\frac{x+2}{x-2}$

$=\frac{-x+4}{x-2}$

f) $\frac{4x^2-20x}{x^2+2x-35}+\frac{3x-6}{x^2-12x+20}$

$=\frac{4x(x-5)}{(x+7)(x-5)}+\frac{3(x-2)}{(x-10)(x-2)}$

$=\frac{4x(x-10)+3(x+7)}{(x+7)(x-10)}$

$=\frac{4x^2-40x+3x+21}{(x+7)(x-10)}$

$=\frac{4x^2-37x+21}{(x+7)(x-10)}$

$x\neq -7,5,10,2$

g) $\frac{a}{a+b} - \frac{b}{b-a} + \frac{2ab}{a^2-b^2}$

$a \neq \pm b$

h) $\frac{2x-6}{x^2-5x+6} - \frac{3x-12}{x^2-x-12}$

$= \frac{2(x-3)}{(x-2)(x-3)} - \frac{3(x-4)}{(x-4)(x+3)}$

$= \frac{2(x+3) - 3(x-2)}{(x-2)(x+3)}$

$= \frac{2x+6-3x+6}{(x-2)(x+3)}$

$= \frac{-x+12}{(x-2)(x+3)}$

$x \neq 2, 3, 4, -3$

i) $\frac{2x}{3x^2-11x+6} - \frac{3x-12}{3x^2-14x+8}$

$x \neq 3, 4, \frac{2}{3}$

$= \frac{2x}{(x-3)(3x-2)} - \frac{3(x-4)}{(x-4)(3x-2)}$

$= \frac{2x-3(x-3)}{(x-3)(3x-2)} = \frac{-x+9}{(x-3)(3x-2)}$

j) $\frac{2x}{3-x} - \frac{3x}{x+3} + \frac{2}{x^2-9}$

$x \neq \pm 3$

$= \frac{-2x}{x-3} - \frac{3x}{x+3} + \frac{2}{(x-3)(x+3)}$

$= \frac{-2x(x+3) - 3x(x-3) + 2}{(x-3)(x+3)}$

$= \frac{-5x^2+3x+2}{(x-3)(x+3)}$

5. Given the equation, find all the possible Non Permissible Values:

$\frac{1}{x + \frac{2}{x+3}}$ $\quad x \neq -3$

$x \neq -3, -1, -2$

$= \frac{1}{\frac{x(x+3)+2}{x+3}} = \frac{x+3}{x^2+3x+2} = \frac{x+3}{(x+1)(x+2)}$

6. Simplify: $\frac{\frac{x-2}{2x} + \frac{1}{x+2}}{\frac{3}{2} - \frac{6}{x^2+2x}}$

$x \neq 0, -2, -1 \pm \sqrt{5}$

$= \frac{(x-2)(x+2)+2x}{3(x^2+2x)-6(2)} = \frac{x^2+2x-4}{3x^2+6x-12} = \frac{x^2+2x-4}{3(x^2+2x-4)} = \frac{1}{3}$

7. How many ordered pairs (a,b), where "a" and "b" are positive integers and a>b, satisfy the equation:

$\frac{1}{a} + \frac{1}{b} = \frac{1}{8}$?

$\frac{b+a}{ab} = \frac{1}{8}$

$8a + 8b = ab$

$8a - ab + 8b = 0$

$a(8-b) = -8b$

$a = \frac{-8b}{8-b}$

Since a & b integers,

$-b > 8$ (because of above)

$-8b$ is a factor of $8b$. So either a factor of 8 or some other factor of b.

$8-b = -1/-2/-4/-8$

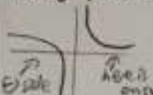
$b = 9/10/12/16$ & $a > b$ so

$a = 72/40/24/16$

So $(72,9)(40,10)(24,12)$

Any more? probably not, can't think of a way to have $8-b$ be a factor of b.

Also, graph of $a = \frac{-8b}{8-b}$



Checking it, no matter seem to work???

So, 3 pairs?

Name: Andy Date: 24/11/20

HW Section 1.5 Solving Rational Functions

1. Indicate which of the following are rational functions. If not, explain why:

a) $y=\frac{x+2}{x-3}$ Yes	b) $y=x^3-x^2$ Yes	c) $y=\frac{2^{x-3}}{x}$ NA (variable in exponent)	d) $y=\frac{\sqrt{x}}{x}$ NA no radicals
e) $y=x^{-3}+\sqrt{2}x^{-2}$ Sure	f) $y=\frac{12}{x}$ why not	g) $y=\lvert x^3+x^2+x\rvert$ Mhm yes	h) x^2+y^2+xy Ye

2. Solve each of the following equations. Indicate any extraneous roots if any:

a) $\frac{4}{x}+\frac{3}{x+2}=5$

$0=(x-1)(5x+8)$

$x=1/-\frac{8}{5}$

$\frac{4(x+2)+3(x)}{x(x+2)}=\frac{5x(x+2)}{x(x+2)}$

$4x+8+3x=5x^2+10x$

$0=5x^2+3x-8$

$\frac{5}{1}\times\frac{-8}{1}$

b) $\frac{-2}{x+3}-\frac{5}{x}=2$

$0=(x+5)(2x+3)$

$x=-5/-\frac{3}{2}$

$\frac{-2(x)}{(x+3)(x)}-\frac{5(x+3)}{x(x+3)}=\frac{2(x)(x+3)}{x(x+3)}$

$-2x-5x-15=2x^2+6x$

$0=2x^2+13x+15$

$\frac{1}{2}\times\frac{15}{1}$

c) $\frac{x^2+6}{3}-\frac{7}{2}=\frac{x+15}{2}$

$0=(x+6)(2x-9)$

$x=-6/\frac{9}{2}$

$\frac{(x^2+6)(2)-7(3)}{6}=\frac{(x+15)(3)}{6}$

$2x^2+12-21=3x+45$

$2x^2-3x-54=0$

$\frac{1}{2}\times\frac{6}{-9}$

d) $\frac{5}{3x-1}-\frac{9}{6x-1}=2$

$0=(3x-2)(12x+1)$

$x=\frac{2}{3}/-\frac{1}{12}$

$\frac{5(6x-1)-9(3x-1)}{(3x-1)(6x-1)}=\frac{2(3x-1)(6x-1)}{(3x-1)(6x-1)}$

$30x-5-27x+9=36x^2-18x+12$

$0=36x^2-21x-2$

$\frac{3}{12}\times\frac{1}{-2}$

e) $\frac{3x}{x-2}+\frac{x}{x+2}=\frac{2x+3}{x+2}$

$\frac{3x(x+2)+x(x-2)}{(x-2)(x+2)}=\frac{2x+3(x-2)}{(x+2)(x-2)}$

$3x^2+6x+x^2-2x=2x^2-4x+3x-6$

$2x^2+5x+6=0$

Uh, no real sols

f) $\frac{2x+3}{x+2}-\frac{x+2}{x-1}=\frac{3x}{x-1}$

$x=-1/-\frac{7}{2}$

$\frac{(2x+3)(x-1)-(x+2)(x+2)}{(x+2)(x-1)}=\frac{3x(x+2)}{(x+2)(x-1)}$

$2x^2-2x+3x-3-x^2-4x-4=3x^2+6x$

$0=2x^2+9x+7$

$\frac{2}{1}\times\frac{7}{1}$

$0=(x+1)(2x+7)$

g) $\frac{3x^2}{x^2-4}-\frac{3x^2}{x^2+5x+6}=\frac{4}{x+3}$

No real sols

$\frac{3x^2}{(x-2)(x+2)}-\frac{3x^2}{(x+2)(x+3)}=\frac{4}{x+3}$

$\frac{3x^2(x+3)-3x^2(x-2)}{(x-2)(x+2)(x+3)}=\frac{4(x-2)(x+2)}{(x-2)(x+2)(x+3)}$

$3x^3+9x^2-3x^3+6x^2=4x^2-16$

$11x^2=-16$

uh...

h) $\frac{3x+1}{x^2-2+x}=\frac{2x-3}{x^2-x-6}-\frac{5}{x^2-4x+3}$

No real sols

$\frac{3x+1}{(x+2)(x-1)}=\frac{2x-3}{(x-3)(x+2)}-\frac{5}{(x-3)(x-1)}$

$\frac{3x+1(x-3)}{(x-3)(x+2)(x-1)}=\frac{(2x-3)(x-1)-5(x+2)}{(x-3)(x+2)(x-1)}$

$3x^2-9x+x-3=2x^2-2x-3x+3-5x-10$

$3x^2-8x-3=2x^2-10x-7$

$x^2+2x+4=0$

3. Solve each of the following equations. Indicate any extraneous roots if any:

a) $\frac{3}{x+2}-\frac{2}{x-1}=5$

$\frac{3(x-1)}{(x+2)(x-1)}-\frac{2(x+2)}{(x+2)(x-1)}=\frac{5(x+2)(x-1)}{(x+2)(x-1)}$

$3x-3-2x-4=5x^2+5x-10$

$x-7=5x^2+5x-10$

$0=5x^2+4x-3$

$x=\frac{-4\pm\sqrt{76}}{10}$

$=\frac{-2\pm\sqrt{19}}{5}$

b) $\frac{2}{x+2}+\frac{1}{x}=1$

$\frac{2x+x+2}{(x+2)(x)}=\frac{(x+2)(x)}{(x+2)(x)}$

$3x+2=x^2+2x$

$0=x^2-x-2$

$0=(x-2)(x+1)$

$x=2/-1$

c) $\frac{2}{y}=\frac{3}{y^2+2}$

$2y^2+4=3y$

$2y^2-3y+4=0$

No real solutions

d) $\frac{x-2}{x-3}+\frac{x-3}{x-2}=\frac{2x^2}{x^2-5x+6}$

$\frac{(x-2)(x-2)+(x-3)(x-3)}{(x-3)(x-2)}=\frac{2x^2}{(x-3)(x-2)}$

$x^2-4x+4+x^2-6x+9=2x^2$

$10x=13$

$x=\frac{13}{10}$

e) $x+\frac{30}{x+8}=3$

$\frac{x(x+8)+30}{x+8}=\frac{3(x+8)}{x+8}$

$x^2+8x+30=3x+24$

$x^2+5x+6=0$

$x=2/3$

f) $\frac{5}{x+1}+\frac{4}{3}=\frac{x+1}{x-1}$

$\frac{5(3)(x-1)+4(x-1)(x+1)}{(x+1)(x-1)(3)}=\frac{(x+1)(x+1)(3)}{(x+1)(x-1)(3)}$

$15x-15+4x^2-4=3x^2+6x+3$

$x^2+9x-22=0$

$(x+11)(x-2)=0$

$x=-11/2$

g) $\frac{2x-1}{2x+1}+\frac{x+1}{x+3}=\frac{3x-1}{2x+1}+\frac{1}{6}$

$\frac{(2x-1)(x+3)(6)+(x+1)(2x+1)(6)}{(2x+1)(x+3)(6)}=\frac{(3x-1)(x+3)(6)}{(2x+1)(x+3)(6)}+\frac{(x+3)(2x+1)}{(2x+1)(x+3)(6)}$

$12x^2+36x-18+12x^2+18x+6=18x^2+48x-18+2x^2+7x+3$

$4x^2-7x+3=0$

$(x-1)(4x-3)=0$

$x=1/3/4$

h) $\frac{2x-3}{x-1}-\frac{x-1}{x+2}=\frac{2x-5}{x+2}+\frac{2-x}{1-x}$

$\frac{2x-3}{x-1}-\frac{x-1}{x+2}=\frac{2x-5}{x+2}+\frac{x-2}{x-1}$

$-\frac{x-2}{x-1}+\frac{2x-3}{x-1}=\frac{2x-5}{x+2}+\frac{x-1}{x+2}$

$\frac{(x-1)(x+2)}{(x-1)(x+2)}=\frac{(2x-6)(x-1)}{(x-1)(x+2)}$

$x^2+x-2=3x^2-9x+6$

$0=2x^2-10x+8$

$0=(x-1)(2x-8)$

$x=4$

Extraneous: $x=1$

4. For which value of "x" will $\frac{3+x}{4+x}$ and $\frac{6+x}{8+x}$ be equal?

$\frac{3+x}{4+x}=\frac{6+x}{8+x}$

OH 4,

$(3+x)(8+x)=(4+x)(6+x)$

$24+11x+x^2=24+10x+x^2$

$11x=10x$

$x=0$

3. Solve each of the following equations. Indicate any extraneous roots if any:

a) $\frac{3}{x+2}-\frac{2}{x-1}=5$

$\frac{3(x-1)}{(x+2)(x-1)}-\frac{2(x+2)}{(x+2)(x-1)}=\frac{5(x+2)(x-1)}{(x+2)(x-1)}$

$3x-3-2x-4=5x^2+5x-10$

$x=\frac{-4\pm\sqrt{76}}{10}$

$0=5x^2+5x-10$

$=\frac{-2\pm\sqrt{19}}{5}$

$0=5x^2+10x-3$

b) $\frac{2}{x+2}+\frac{1}{x}=1$

$\frac{2x+x+2}{(x+2)(x)}=\frac{(x+2)(x)}{(x+2)(x)}$

$3x+2=x^2+2x$

$0=x^2-x-2$

$0=(x-2)(x+1)$

$x=2/-1$

c) $\frac{2}{y}=\frac{3}{y^2+2}$

$2y^2+4=3y$

$2y^2-3y+4=0$

No real solutions

d) $\frac{x-2}{x-3}+\frac{x-3}{x-2}=\frac{2x^2}{x^2-5x+6}$

$\frac{(x-2)(x-2)+(x-3)(x-3)}{(x-3)(x-2)}=\frac{2x^2}{(x-3)(x-2)}$

$x^2-4x+4+x^2-6x+9=2x^2$

$10x=13$

$x=13/10$

e) $x+\frac{30}{x+8}=3$

$\frac{x(x+8)+30}{x+8}=\frac{3(x+8)}{x+8}$

$x^2+8x+30=3x+24$

$x^2+5x+6=0$

$x=2/3$

f) $\frac{5}{x+1}+\frac{4}{3}=\frac{x+1}{x-1}$

$\frac{5(3)(x-1)+4(x-1)(x+1)}{(x+1)(3)(x-1)}=\frac{(x+1)(x+1)(3)}{(x+1)(x-1)(3)}$

$15x-15+4x^2-4=3x^2+6x+3$

$x^2+9x-22=0$

$(x+11)(x-2)=0$

$x=-11/2$

g) $\frac{2x-1}{2x+1}+\frac{x+1}{x+3}=\frac{3x-1}{2x+1}+\frac{1}{6}$

$\frac{(2x-1)(x+3)(6)+(x+1)(2x+1)(6)}{(2x+1)(x+3)(6)}=\frac{3x-1(x+3)(6)}{2x+1(x+3)(6)}+\frac{(x+3)(2x+1)}{(2x+1)(x+3)(6)}$

$12x^2+30x-36+12x^2+18x+6=18x^2+48x-18+2x^2+7x+3$

$4x^2-7x+3=0$

$(x-1)(4x-3)=0$

$x=1/3/4$

h) $\frac{2x-3}{x-1}-\frac{x-1}{x+2}=\frac{2x-5}{x+2}+\frac{2-x}{1-x}$

$\frac{2x-3}{x-1}-\frac{x-1}{x+2}=\frac{2x-5}{x+2}+\frac{x-2}{x-1}$

$-\frac{x-2}{x-1}+\frac{2x-3}{x-1}=\frac{2x-5}{x+2}+\frac{x-1}{x+2}$

$\frac{(x-1)(x+2)}{(x-1)(x+2)}=\frac{(2x-6)(x-1)}{(x-1)(x+2)}$

$x^2+x-2=2x^2-8x+6$

$0=2x^2-10x+8$

$0=(x-1)(2x-8)$

$x=4$

Extraneous: $x=1$

4. For which value of "x" will $\frac{3+x}{4+x}$ and $\frac{6+x}{8+x}$ be equal?

$\frac{3+x}{4+x}=\frac{6+x}{8+x}$

OH ok

$(3+x)(8+x)=(4+x)(6+x)$

$24+11x+x^2=24+10x+x^2$

$11x=10x$

$x=0$

5. The rational expression $\frac{2x^2+1}{x^2-3}$ may be written as $2+\frac{A}{x^2-3}$, where "A" is an integer. What is the value of "A"?

$\frac{2x^2+1}{x^2-3} = \frac{2(x^2-3)}{x^2-3} + \frac{A}{x^2-3}$ $\quad A=7$

$2x^2+1 = 2x^2-6+A$

6. Sovle for "k" $\frac{3}{x-1}+\frac{k}{x}+\frac{7}{x+1}=\frac{5x^2-4x+5}{x^3-x}$

$\frac{3(x)(x+1)+k(x-1)(x+1)+7(x)(x-1)}{(x-1)(x)(x+1)} = \frac{5x^2-4x+5}{x(x-1)(x+1)}$

$3x^2+3x+kx^2-k+7x^2-7x = 5x^2-4x+5$

$kx^2-k = -5x^2+5$

$k=-5$

7. Solve for "x": $1+\frac{1}{1+\frac{1}{x}+\frac{1}{2x}}=\frac{7}{5}$

$\frac{2}{5} = \frac{1}{\frac{2x+2+1}{2x}}$ $\quad 4x+6=10x$

$\frac{2}{5} = \frac{2x}{2x+3}$ $\quad x=1$

8. For what value(s) of "x" is the equation true? $\frac{8}{9}=\frac{x}{x+\frac{x}{x+x}}$

$\frac{8}{9} = \frac{x}{x+\frac{x}{2x}}$ $\quad 8(2x+1)=18x$

$16x+8=18x$

$= \frac{x}{\frac{2x+1}{2}}$ $\quad x=4$

$8/9 = \frac{2x}{2x+1}$

9. Express "k" as a common fraction in terms of "n" $\frac{k(n-2)!}{(n+1)!}=\frac{(n-1)!}{(n+2)!}$

$\frac{k(n-2)!}{(n+1)(n)(n-1)(n-2)!} = \frac{(n-1)!}{(n+2)(n+1)(n)(n-1)!}$

$\frac{k}{(n+1)(n)(n-1)} = \frac{1}{n+2)(n+1)(n)}$

$k=\frac{n-1}{n+2}$

10. What is the greatest integer "n" for which $\frac{24n}{n-4}$ is an integer?

$\frac{24n}{n-4}=k$

$24n = kn-4k$

$4k = n(k-24)$

$n=\frac{4k}{k-24}$

Since everything is integral, $n=\frac{4k}{k-24}$ is also integral.

Let's just assume 0 integers too. (we want n to be BIG ok?)

Minimize denominator $\rightarrow k=25$

$n=\frac{100}{1}=100$

So $n=100$

Also, graph $\frac{24n}{n-4}$ do the something * minimize down like top doesn't exist but the bottom does.

11. Solve the following function for "k": $\dfrac{1}{1-\dfrac{1}{1-\dfrac{1}{1-\dfrac{1}{x}}}} = x^k$

$k = -1$

12. For all values other than $x=-1$ and $x=2$, what is the value of $\dfrac{2x^2-x}{x^2-x-2} - \dfrac{4+x}{(x+1)(x-2)}$?

$= 2$

13. Evaluate: $\displaystyle\sum_{x=1}^{2003} \frac{1}{x^2+7x+12}$

$= \dfrac{2003}{8028}$

14. If "a" and "b" are positive integers such that $\dfrac{1}{a}+\dfrac{1}{2a}+\dfrac{1}{3a} = \dfrac{1}{b^2-2b}$, then which of the following is the smallest possible value of $a+b$?

a) 8 b) 6 c) 96 d) 10 e) 50

$a=44$, $b=6$ works!

So $a+b=50$

15. Challenge: Given that "n" is an integer, for how many values of "n" is $\dfrac{2n^2-10n-4}{n^2-4n+3}$ an integer?

$n=-5$, $n=2$, $n=4$ and $n=7$ work

So, 5

Name: Andy Date: 25/11/20

HW Section 1.6 Applications of Rational Functions

1. Jason can fill a water tank in 4 min using a large hose. He takes 6 min using a smaller hose. How long will he take if he uses both hoses?

$\frac{x}{4} + \frac{x}{6} = 1$ $\quad x = \frac{24}{10} = \frac{12}{5}$ min

$\frac{10x}{24} = 1$

2. Mario can take inventory at the store in 30min. His partner, Carmen, can take inventory in 20min. If they work together, how long will the inventory take?

$\frac{x}{30} + \frac{x}{20} = 1$ $\quad x = 12$ min

$\frac{50x}{600} = 1$

3. Andrew can deliver 500 handbills in 2h. Amos can deliver the same number in 3h. How long will they take to deliver 500 handbills if they work together?

Let 1 = 500 handbills

$\frac{x}{2} + \frac{x}{3} = 1$ $\quad \frac{5x}{6} = 1$ $\quad x = \frac{6}{5}$ hr

4. Suppose Andrew can deliver 300 wedding cards in 2h. Susan can deliver the 400 cards number in 3h. How long will they take to deliver 1000 cards if they work together?

$\frac{300}{2}x + \frac{400}{3}x = 1000$ $\quad x = \frac{300}{85}$

$\frac{850}{3}x = 1000$ $\quad = \frac{60}{17}$ hr

5. On a highway between Vancouver to Kelowna, the speed limit is 90km/hr and the distance is 390km. Most cars take about 4 hours and 20minutes. Some marks travel faster than 90km/hr and some slower, taking either more or less time respectively.

   a. If someone travels at 120km/hr, how much time will they save? ~~At what speed~~

$\frac{1339}{420} - 4\frac{1}{3}$ $\quad = -\frac{13}{12}$ $\quad$ They save $\frac{13}{12}$ hrs.

$= \frac{13}{4} - \frac{13}{3}$

   b. At what speed does it take to make the trip in 4hours?

$4 = \frac{390}{x}$ $\quad = \frac{195}{2}$ km/hr

$x = \frac{390}{4}$ 195 2

6. The average cost "A" dollars, of printing the school agenda is given by the equation: $A = \frac{2500 + 1.25n}{n}$, where "n" is the number printed.

   a. Determine the average cost when 500 agendas are printed

$A = \frac{2500 + 1.25 \cdot 500}{500}$

$= 5 + \frac{1.25 \cdot 500}{500}$

$= \$6.25$

   b. Determine the number of agendas are printed when the average cost is $8.00?

$8 = \frac{2500 + 1.25n}{n}$

$6.75n = 2500$

$n = \frac{10000}{27}$

≈ 370 agendas

(370, 370)

7. A tour boat's top speed is 40km/h on calm water. The boat leaves its dock and goes straight upstream 15km to see a waterfall. It then returns downstream to it's dock. On both trips the boat travels on full speed. Suppose the current is 25km/h, what is the time needed for a round trip of the tour?

	D	S	T
To	15	15	?
Back	15	65	

$T = \frac{15}{15} + \frac{15}{65}$

$= \frac{16}{13}$ hr

8. Amy flies her plane each week from Lethbridge to Moose Jaw. On both trips, the air speed is 180km/h. On the flight out there is a constant tail wind of 20km/hr and on the return trip with a head wind of the same speed. Calculate the time needed for a round trip.

	D	S	T
To	526 km (approx)	200	
Back		160	

$T = \frac{526}{200} + \frac{526}{160} = 5.9175$hr

9. Suppose Amy is the pilot and she needs to make the round trip in 5hours, what speed would she need to fly? Suppose the stopover time is very short.

$5 = \frac{526}{x+20} + \frac{526}{x-20}$

$5 = \frac{526(x-20) + 526(x+20)}{x^2 - 400}$

$5x^2 - 2000 = 1052x$

$5x^2 - 1052x - 2000 = 0$

$x \approx \frac{1052 \pm 1032}{10}$

$x = 208.4 / 2$

Speed is ~~2km/hr~~ 208.4km/hr

10. On the way from Vancouver to Seattle, the speed limit is 110km/h. Since the distance between the cities is approximately 230km, a trip between the two cities is about 2.1hours (Excluding border wait time). Cars travelling faster can reach their destination within 2.1 hours.

a. Let "s" represent the change in speed compared with 110km/h. Let "t" represent the change in time compared with 2.1hrs. Write "t" as a function of "s"

$t = \frac{230}{110} - \frac{230}{(110+s)}$

$t = 2.1 - \frac{230}{110+s}$

b. How much time will you save driving at 125km/h?

$t = 2.1 - \frac{230}{125}$

$= \frac{13}{90}\text{hr}$

c. At what speed does it take to save 10min?

$\frac{1}{6} = 2.1 - \frac{230}{s}$

$\frac{29}{15} = \frac{230}{s}$

$s = \frac{3450}{29}\text{km/hr}$

11. The average speed of a car is three times as fast as the average speed of a cyclist. To travel 225km, the cyclist requires 5h more than the car. Determine the average speed of both the cyclist and the car.

	D	S	T
C	225	$3x$	$\frac{225}{3x}$
B		x	$\frac{225}{x}$

$\frac{225}{3x} + 5 = \frac{225}{x}$

$\frac{225 + 15x}{3x} = \frac{675}{3x}$

Bike $x = 30$ km/hr
Car $= 90$ km/hr

12. An plane flies 900km with a tailwind and then returns the same distance against the wind. The round trip lasts 7 hours. If the plane's speed in still air is 280km/hr, what is the speed of the wind?

$\frac{900}{280-x} + \frac{900}{280+x} = 7$

$\frac{900(280+x) + 900(280-x)}{280^2 - x^2} = 7$

$504000 = 548800 - 7x^2$

$x^2 = 6400$

$x = 80$ km/hr

13.

14. An airplane travels between two cities that are 350 km apart. One day the airplane leaves 15 minutes behind schedule. In order to arrive at its destination on time, the airplane flies 25km/h faster than usual. What is the airplane's speed?

	D	S	T
Usual	350	x	$\frac{350}{x}$
Late		$x+25$	$\frac{350}{x+25}$

$\frac{350}{x+25} + \frac{1}{4} = \frac{350}{x}$

$\frac{1400 + x + 25}{4(x+25)} = \frac{350}{x}$

$1400x + x^2 + 25x = 1400x + 35000$

$x^2 + 25x - 35000 = 0$

$x = \frac{-25 \pm 375}{2}$

$x = 175/\!\!\cancel{-200}$

So speed now $= 175 + 25 = 200$ km/hr

15. Emily can row 10km/hr in still water. One day it took her 4 hours longer to go a certain distance upstream than the same distance downstream. If the speed of the current was 5km/hr, how far upstream did she go?

	D	S	T
Up	x	5	$\frac{x}{5}$
Down	x	15	$\frac{x}{15}$

$\frac{x}{15} + 4 = \frac{x}{5}$

$4 = \frac{3x}{15} - \frac{x}{15}$

$4 = \frac{2x}{15}$

$x = 30$km

16. A group of hikers need to travel 80km from Camp A to Camp B and then 128km from Camp B to Camp C. It takes two days more to travel from B to C than from A to B. The average speed from A to B is also 3km more than the average speed from B to C. What was the average speed of travel from Camp A to B?

	D	S	T
A→B	80	$x+3$	$\frac{80}{x+3}$
B→C	128	x	$\frac{128}{x}$

$x =$ km/day

$\frac{80}{x+3} + 2 = \frac{128}{x}$

$\frac{80 + 2x + 6}{x+3} = \frac{128}{x}$

$2x^2 + 86x = 128x + 384$

$2x^2 - 42x - 384 = 0$

$x^2 - 21x - 192 = 0$

$x = \frac{21 \pm \sqrt{1089}}{2}$

So A→B $= \frac{27 + \sqrt{1089}}{2}$ km/day

17. Tim and Hank paint a fence for four hours, after which Jack helps them and they finish two hours later. If jack had not helped them, it would have taken them five more hours to paint the fence. How long would it take for Tim to paint the fence alone?

$\frac{6}{(T+H)} + \frac{2}{J} = 1$

$\frac{9}{(T+H)} = 1 \rightarrow (T+H) = 9$

$\frac{6}{9} + \frac{2}{J} = 1$

$J = 6$hr

Uh, not enough info to find Tim's speed...

So uh

oh no.

Jack takes 6hr to paint that fence.